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On the simplified calculation of multi-box deck flutter derivatives

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SUMMARY:
An enhancement of a method of literature for the approximate calculation of multi-box girder flutter derivatives is proposed. The original method is based on the linear superposition of flat plate flutter derivatives, where the plates width and distance from the elastic axis are embedded in the formulation. In the modified version proposed here, the flutter derivatives of each box are calculated according to Theodorsen's flat plate theory, wherein the steady state aerodynamic coefficients of the flat plate are replaced by those of the actual boxes. The lift and moment coefficients of the different boxes are calculated by static CFD analyses of the whole multi-box section. In this way, the wind flow variation induced by the upstream bodies is taken into account, albeit in a static manner. Three different cross-sections are investigated, for which the computed flutter derivatives are compared to the experimental ones, showing improvements with respect to those predicted by the original simplified method. In particular, a noticeable improvement is observed for the evaluation of the aerodynamic torsional stiffness. Eventually, the proposed formulation is tested for the flutter prediction of the 1915 Çanakkale Bridge, showing that the computed flutter derivatives provide a correct assessment of the coupled mode flutter.

Keywords: multi-box decks, flutter derivatives, long-span bridges

1. INTRODUCTION

Multi-box girders have proven to be a good solution for super-long-span suspension bridges, meeting the mass reduction and aerodynamic performance requirements (Diana et al., 2003). Some remarkable examples are the actual main span record 1915 Çanakkale Bridge in Turkey and the Xihoumen Bridge in China. The aerodynamic performance of such girders has been widely investigated over the past decades: among the various peculiarities, experimental observations emphasized the gap-width influence on the aerodynamic damping (Qin et al., 2007) and the aerodynamic stiffness affecting the interaction between the degrees of freedom (Yang, Wu, et al., 2015; Yang, Zhou, et al., 2015). Recently, (Andersen et al., 2022) introduced an analytic expression of flutter derivatives for an arbitrary multi-box girder based on the superposition of flat plate flutter derivatives. The formulation introduced allows to take into account the distance of each single box from the elastic axis as well as the different box widths on the estimation of flutter derivatives.

In this work, an enhancement of the former method is proposed accounting for the aerodynamic interaction between different boxes by means of their lift and moment coefficients. The connection between steady-state aerodynamic coefficients, Theodorsen's functions, and flutter derivatives is well detailed in (de Miranda et al., 2013). A recent alternative way to express flutter derivatives of a twin-box girder by means of steady state lift and moment slopes can be found in (Ronne et al., 2021). Here, three different deck cross-sections made of two or three streamlined boxes are considered. Results show that the flutter derivatives calculated by the proposed method are in good agreement with the experimental ones. In particular, the introduced formulation allows to accurately predict the aerodynamic torsional stiffness involved in the coupled-mode flutter instability. As a further validation, a MDOF flutter analysis of the 1915 Çanakkale bridge is performed by an *in-house* Matlab code. A comparison is thus made between the flutter critical parameters obtained using the experimental flutter derivatives (Liao et al., 2022) and those predicted by the proposed analytical method.

2. METHODS

Following (Andersen et al., 2022), flutter derivatives of a generic multi-box cross-section can be approximately estimated by the superposition of flutter derivatives of the different boxes. Here, the latter are calculated by the flat plate theory, modified by including the derivatives of the static aerodynamic coefficients. In this way, it is assumed that the air circulation around each box and the resulting wake configuration affect only the lift and moment variations with respect to the wind angle of attack. Under these assumptions, flutter derivatives of an *n*-box girder cross-section are given by:

$$H_1^* = -\frac{1}{k_T b_T}\sum_{j=1}^{n}\left[C'_{l\,j} b_j F(k_j)\right] \tag{1a}$$

$$H_2^* = -\frac{1}{2k_T * b_T^2}\sum_{j=1}^{n}\left[C'_{l\,j} b_j^2\left(1 + (1-2a_j)F(k_j) + 2\frac{G(k_j)}{k_j}\right)\right] \tag{1b}$$

$$H_3^* = -\frac{1}{2k_T^2 b_T}\sum_{j=1}^{n}\left[C'_{l\,j} b_j\left(a_j k_j^2 + 2F(k_j) - (1-2a_j)G(k_j)k_j\right)\right] \tag{1c}$$

$$H_4^* = -\frac{1}{k_T^2}\sum_{j=1}^{n}\left[C'_{l\,j} b_j F(k_j)\right] \tag{1d}$$

$$A_1^* = \frac{4}{k_T b_T^2}\sum_{j=1}^{n}\left[C'_{m\,j} b_j^2 F(k_j)(\tfrac{1}{2} + a_j)\right] \tag{1e}$$

$$A_2^* = -\frac{2}{k_T b_T^3}\sum_{j=1}^{n}\left[C'_{m\,j} b_j^3\left(k_j^2\left(\frac{1}{2} - a_j\right) + 2F(k_j)\left(a_j^2 - \frac{1}{5}\right) - \frac{2G(k_j)}{k_j}\left(a_j + \frac{1}{2}\right)\right)\right] \tag{1f}$$

$$A_3^* = \frac{2}{k_T^2 b_T^2}\sum_{j=1}^{n}\left[C'_{m\,j} b_j^2\left(k_j^2\left(a_j^2 + \frac{1}{8}\right) + 2F(k_j)\left(a_j + \frac{1}{2}\right) + 2k_j G(k_j)(a_j^2 - \tfrac{1}{5})\right)\right] \tag{1g}$$

$$A_4^* = -\frac{2}{k_T^2}\sum_{j=1}^{n}\left[C'_{m\,j} b_j\left(a_j k_j^2 + 2a_j G(k_j)(a_j + \tfrac{1}{2})\right)\right] \tag{1h}$$

where b_T is the sum of the box half-widths, $k_T = \omega b_T / U$ is the reduced frequency of the whole cross-section, being ω the angular frequency and U the wind velocity. The terms with the subscript *j* refer to the *j*-th box: $C'_{l\,j}$ and $C'_{m\,j}$ are the derivatives of lift and moment coefficients with respect to the wind angle of attack (here calculated around the null angle); b_j is the half-width; a_j is the non-dimensional distance between the box chord midpoint and the global rotation axis (elastic axis); $k_j = \omega b_j / U$ is the box reduced frequency; $F(k_j)$ and $G(k_j)$ are the real and the imaginary parts of Theodorsen's complex circulatory function.

Other simplified formulations were derived from the general expressions of Eqs. (1a-h) and investigated for the sake of comparison. Their characteristics are summarized in Table 1.

Table 1. Simplified formulations adopted for the prediction of multi-box deck flutter derivatives.

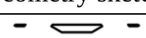
Method	Aerodynamics	*n* in Eqs. (1)	Lift and moment coefficients' slope in Eqs. (1)
1	Flat plate	1	Thin airfoil
2	Flat plate modified	1	Calculated by CFD for the whole cross-section
3	Superposition of flat plates	*n*	Thin airfoil
4	Superposition of flat plates modified	*n*	Calculated by CFD for each box in the assembly

Steady-state aerodynamic coefficients of different boxes were calculated in Ansys Fluent by static CFD analyses of the multi-box system, variating the wind angle of attack between –5° and +5°. RANS method coupled with the $k-\omega\ SST$ model was used to model turbulence. Flutter analyses were performed by an *in-house* Matlab code based on a modal Galerkin discretization of an analytic continuum model of suspension bridges.

3. RESULTS

The method proposed was applied to three case studies (Table 2): two triple-box girders with different gap-widths analyzed in (Andersen et al., 2022) (series 6 and 7), where the experimental flutter derivatives are also reported, and the twin-box girder of the 1915 Çanakkale Bridge, of which experimental flutter derivatives are reported in (Liao et al., 2022).

Table 2. Case studies (widths *b* are in mm).

Case	Typology	Geometry sketch	b_T	b_1	b_2	b_3	a_1	a_2	a_3
1	Triple-box		230	35	160	35	10.14	0	–10.14
2	Triple-box		230	35	160	35	14.71	0	–14.71
3	Twin-box		180.3	90. 15	90.15	-	1.442	–1.442	-

As an example, Fig. 1 shows the results for A_2^* and A_3^* flutter derivatives. The predictions furnished by the proposed method (Method 4) are comparable or better than those of the other three methods. The discrepancy in the derivative A_2^* is significant for Case 2 and larger for Case 3. Nevertheless, in this latter case, Method 4 provides the best prediction of A_2^* in comparison with the other methods analyzed here (Fig. 1).

Lastly, a MDOF flutter analysis was performed for the 1915 Çanakkale Bridge bridge, adopting the experimental flutter derivatives and the ones calculated by the four methods described in Table 1. The resulting critical wind speeds and frequencies are summarized in Table 3.

Table 3. Flutter critical parameters for the 1915 Çanakkale Bridge adopting different flutter derivatives.

	Method 1	Method 2	Method 3	Method 4	Exp. flutter deriv.
Flutter wind speed (m/s)	91.15	115.02	126.28	88.69	87.78
Flutter frequency (Hz)	0.114	0.109	0.117	0.102	0.097

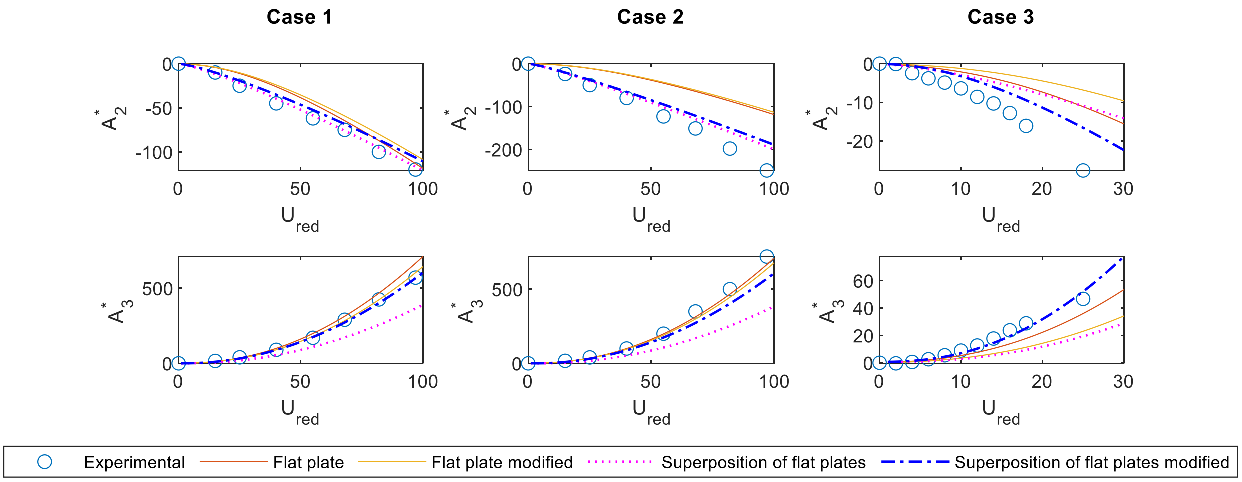


Figure 1. Flutter derivatives from different approximate methods vs. experimental values.

The comparison shows that Methods 1, 2 and 3 provide unsafe predictions. Method 4 provides a good agreement with the analysis based on measured flutter derivatives.

4. CONCLUSIONS

An analytic formulation for the approximate calculation of multi-box girder flutter derivatives based on the linear superposition of modified expressions of flat plate flutter derivatives was presented. The comparison with experimental flutter derivatives of streamlined multi-box sections showed in general a reasonable agreement, although with some discrepancies. The proposed method, due to the simplification introduced, cannot explain comprehensively the complex aerodynamic interaction between different boxes but can provide a useful tool for investigating some peculiar features of multi-box girders such as stiffness-driven flutter instability. The obtained results encourage further investigations.

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